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The Combined Effects of Pressure Gradient and Heating on the Stability and Transition of Boundary Layers in Water

A. R. Wazzan, C. Gazley, Jr.

A Report prepared for
DEFENSE ADVANCED RESEARCH PROJECTS AGENCY

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PREFACE

Under the sponsorship of the Tactical Technology Office of the Defense Advanced Research Projects Agency, The Rand Corporation has been developing hydrodynamic design criteria employing concepts of boundary-layer control.

This report presents the results of numerical computations and analyses of boundary-layer stability and predicted transition for a series of heated wedge flows having both favorable and adverse pressure gradients. The results are generalized in a form that is useful for the conceptual design of axisymmetric bodies that employ combinations of heating and shaping to achieve an extended region of laminar flow. Preliminary versions of these results were presented at the Second Low-Speed Boundary-Layer Transition Workshop in Santa Monica, California, September 13-15, 1976, and at the Second International Conference on Drag Reduction in Cambridge, England, August 31-September 2, 1977.

The report should be useful to hydrodynamicists, designers of submersibles, and others engaged in applying fluid mechanics methodology to the improvement of underwater vehicle performance. Related Rand reports include:

R-1752-ARPA/ONR, *Low-Speed Boundary-Layer Transition Workshop*, W. S. King, June 1975.

R-1789-ARPA, *Controlling the Separation of Laminar Boundary Layers in Water: Heating and Suction*, J. Aroesty and S. A. Berger, September 1975.

R-1863-ARPA, *The Effects of Wall Temperature and Suction on Laminar Boundary-Layer Stability*, W. S. King, April 1976.

R-1898-ARPA, " e^9 " *Stability Theory and Boundary-Layer Transition*, S. A. Berger and J. Aroesty, February 1977.

R-1966-ARPA, *The Buoyancy and Variable Viscosity Effects on a Water Laminar Boundary Layer Along a Heated Longitudinal Horizontal Cylinder*, L. S. Yao and Ivan Catton, February 1977.

R-2111-ARPA, *Entry Flow in a Heated Tube*, L. S. Yao, June 1977.

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R-2164-ARPA, *The Effects of Unsteady Potential Flow on Heated Laminar Boundary Layers in Water: Flow Properties and Stability*, W. S. King, J. Aroesty, L. S. Yao, and W. Matyskiela, November 1977.

R-2165-ARPA, *Approximate Methods for Calculating the Properties of Heated Laminar Boundary Layers in Water. Part I: Constant Surface Temperature*, G. M. Harpole, S. A. Berger, and J. Aroesty, January 1978.

R-2209-ARPA, *Simple Relations for the Stability of Heated Laminar Boundary Layers in Water: Modified Dunn-Lin Method*, J. Aroesty, W. S. King, G. M. Harpole, W. Matyskiela, A. R. Wazzan, and C. Gazley, Jr. (in process).

SUMMARY

Appreciable drag reduction is possible if extended regions of laminar flow can be maintained. Although a variety of techniques for boundary-layer control have been explored, only recently has the effect of heat transfer on the stability and transition of water boundary layers been investigated. In spite of an early experiment⁽¹⁾ which did not indicate any favorable effects of heating on the stability of a water boundary layer in a tube, speculation continued^(2,3,4) that the heating of water boundary layers might increase their stability because of the large variation of viscosity with temperature. An approximate analysis for flat-plate flow⁽⁵⁾ and numerical computations^(6,7,8) have confirmed the increase in stability for stagnation, flat-plate, and separating flows. Spatial amplification computations⁽⁹⁾ also indicate an appreciable effect of heating on boundary-layer transition for flat-plate flow.

This study presents additional computations for the stability and predicted transition characteristics of water boundary-layer "wedge" flows for Hartree β 's ranging from -0.15 to +0.40 and surface temperatures up to 67°C (120°F) above the ambient temperature. Both the minimum critical Reynolds number and the predicted transition Reynolds number of these "similar" boundary layers increase as the surface temperature is increased above the ambient level.

The interacting effects of pressure gradient and surface heating on stability and predicted transition may be approximately characterized by a boundary-layer shape parameter such as $H = \delta^*/\theta$. The computed distance Reynolds numbers for neutral stability and predicted transition are given as a function of H . Although this stability-transition map has been formed by computations for similar boundary layers, it can usefully be employed in the analysis of heated bodies with nonsimilar boundary-layer development. Several examples are presented which show that for near similar flows this map is quite accurate; for an extreme departure from similarity the boundary-layer history must

be considered. In order to maintain an extended region of laminar flow, it is apparent that the boundary-layer development should follow a path in which the shape parameter is kept as low as possible over as great a range of Reynolds number as possible.

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NOMENCLATURE

A = amplitude of disturbance
a = amplification ratio
c = disturbance velocity
 c_{fL} = local laminar skin-friction coefficient
 c_p = specific heat
H = boundary-layer shape parameter = δ^*/θ
h = heat-transfer coefficient
k = thermal conductivity
L = characteristic length
Nu = Nusselt number = hx/k
n = e exponent, natural logarithm of amplification ratio
Pr = Prandtl number = $c_p \mu/k$
R = Reynolds number based on boundary-layer thickness $U_e \delta/\nu$
 R_L = freestream length Reynolds number = $U_\infty L/\nu_\infty$
Re = Reynolds number = $U_e x/\nu$
t = percent freestream turbulence level, also time
T = temperature
u, U = velocity in x direction
v = velocity in y direction
 v', u' = velocity fluctuations
x = distance in flow direction
y = distance normal to wall
 α_1 = spatial amplification rate
 β = Hartree parameter
 δ = boundary-layer thickness
 δ^* = boundary-layer displacement thickness
 μ = viscosity
 ν = kinematic viscosity
 ω = dimensionless frequency = $\omega^* \nu/U_\infty^2$
 ω^* = dimensional frequency
 ψ = stream function
 ϕ = disturbance velocity amplitude

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τ = shear stress
 τ_L = laminar shear stress
 θ = boundary-layer momentum thickness
 Θ = disturbance temperature amplitude

Subscripts

e = condition at edge of boundary layer
w = wall condition
x = conditions at station x
 ∞ = freestream conditions
 δ^* = based on boundary-layer displacement thickness
(crit) = neutral stability
(e^9) = condition for amplification ratio = e^9
TS = Tollmien-Schlichting waves

I. INTRODUCTION

The design of high performance hydrodynamic and aerodynamic bodies requires among other factors a knowledge of the process of transition from laminar to turbulent flow and of the dependence of transition on the pressure distribution, the surface temperature, the surface roughness, freestream turbulence, etc. When surface roughness is not a factor and the freestream turbulence level, t , is very low, say $t = u'/u_\infty \leq 0.2\%$, transition in boundary-layer flows results from the growth of Tollmien-Schlichting waves. The growth of Tollmien-Schlichting waves, in flows where the parallel flow assumptions are reasonably valid, can be determined from linear instability theory. That the theory of small disturbances is capable of predicting the growth (or decay, etc.) of Tollmien-Schlichting waves has been demonstrated as early as 1942 by the well-known measurements of Schubauer and Skramstad⁽¹⁰⁾ and most recently by the measurements of Rogler and Reshotko.⁽¹¹⁾ It has been demonstrated that under these conditions, the transition Reynolds number, or bounds on the transition Reynolds number, can be predicted using linear instability theory (Liepmann,⁽¹²⁾ Smith and Gamberoni,⁽¹³⁾ Jaffe et al.⁽¹⁴⁾). The premise behind these forecasting techniques is that the flow remains laminar until the disturbance amplitude (the Tollmien-Schlichting wave with the most dangerous frequency) is amplified by a factor e^n where $7.0 \leq n \leq 10$. Recently, Mack⁽¹⁵⁾ showed that the well-known flat-plate transition results of Dryden can be predicted from linear instability theory using an amplification ratio e^n where

$$n = -8.43 - 2.4 \ln \frac{t}{100}$$

where t is the percent turbulence level; the fit is best in the range of $.08 < t < 1.0$. It is to be expected that the total amplification ratio at transition is lower for higher freestream turbulence levels if we assume that transition occurs when u'/u_∞ reaches a fixed level, say 4 percent. Within the framework of linear instability theory, disturbances begin to amplify once the critical Reynolds number

$Re_{x(crit)}$ is reached. The process terminates, transition occurs, at Re_x , where the initial disturbance amplitude is amplified by a factor of e^n ; let us denote this Reynolds number by $Re_{x(e^9)}$. Within linear instability theory, $Re_{x(crit)}$ for a body of revolution, e.g., is directly dependent on the local boundary layer, which is determined by the local pressure gradient, the body shape, and the surface boundary conditions, e.g., surface heating, cooling, suction, blowing, etc. On the other hand, $Re_{x(e^9)}$, which is the result of integrated spatial amplification rates, is dependent both on the local critical Reynolds number and on the width of the neutral stability curve at *each* station; that is to say, $Re_{x(e^9)}$ is dependent on the local boundary layer as well as on the history of the boundary-layer development.

If the characteristics of the local boundary layer can be well represented by the shape parameter $H(x) \equiv \delta^*/\theta$, the variation of H with Re_x for a body of revolution, e.g., may well serve to determine the stability of the boundary layer to small disturbances if a relationship does exist between H and $Re_{x(crit)}$ and/or $Re_{x(transition)}$. Relationships between H (or U'' and β , which are related to H) and Re_x do exist for certain boundary layer flows, for example, two-dimensional wedge flows. (7,9,16,17) However, relationships between H and $Re_{x(transition)}$ are not yet available in the literature. Such a relationship would be an invaluable tool in the design of high performance hydrodynamic and aerodynamic bodies; the development of such a relationship is the object of this study. In arriving at this relationship, it is assumed that

$$Re_{x(transition)} \equiv Re_{x(e^9)}.$$

Second, it is also assumed that the stability characteristics of a body of revolution at a given station can be approximated by the stability characteristics of a two-dimensional wedge flow with the same pressure gradient as that of the body of revolution at the station in question. With these two assumptions in mind, we proceed then to formulate (compute) a relationship between $Re_{x(crit)}$ and $H(Re_x)$ and a second relationship between $Re_{x(e^9)}$ and $H(Re_x)$ for adiabatic and heated two-dimensional wedge flows. This task requires knowledge of (1) the

boundary layer on adiabatic and heated two-dimensional wedge flows and (2) the stability characteristics (e.g., critical Reynolds number, spatial amplification rates, etc.) of those boundary layers. This information is also given in this report.

II. ANALYSIS

DETERMINATION OF THE MEAN FLOW

The mean flow profiles for heated wedge flows in water with $T_e = 19.4^\circ\text{C}$ (67°F) and $19.4^\circ\text{C} \leq T_w \leq 86^\circ\text{C}$ ($67^\circ \leq T_w \leq 187^\circ\text{F}$) have been calculated as outlined by Kaups and Smith.⁽²⁾ In these calculations all fluid properties are allowed to vary with temperature only (a good assumption for water boundary layers at moderate pressures). In cases where buoyancy effects are not important, viscosity is found to be the most important variable fluid property (Fig. 1). Therefore, in formulating the stability problem, only viscosity variations with temperature have been taken into account. The variation of the following boundary-layer characteristics $(\delta^*/x)\sqrt{\text{Re}_x}$, $(\theta/x)\sqrt{\text{Re}_x}$, $(c_f/2)\sqrt{\text{Re}_x}$, and $\text{Nu}/(\sqrt{\text{Re}_x} \text{Pr}^{1/3})$ with $(T_w - T_e)$ are shown in Fig. 2 and the variation of $H = \delta^*/\theta$ with $(T_w - T_e)$ is shown in Fig. 3.

FORMULATION OF THE LINEAR STABILITY PROBLEM

Neglecting temperature fluctuations, assuming viscosity is a function of temperature only, and taking all other fluid properties constant, Wazzan et al.⁽⁶⁾ found the linearized parallel flow stability problem of water boundary layers with heat transfer can be adequately treated by solving the Orr-Sommerfeld equation modified to include the variation of viscosity with temperature:

$$\begin{aligned} (U - c)(\phi'' - \alpha^2\phi) - U''\phi + \frac{1}{\alpha R} [\mu(\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi) \\ + 2\mu'(\phi''' - \alpha^2\phi') + \mu''(\phi'' + \alpha^2\phi)] = 0 \end{aligned} \quad (1)$$

All quantities in Eq. (1) are dimensionless where the reference values for velocity, length, and viscosity are the edge velocity, U_e , the boundary-layer thickness δ , and the edge viscosity μ_e . R is the Reynolds number based on δ , $R = U_e\delta/\nu_e$. The prime indicates differentiation with respect to y where $y \equiv y^*/\delta$ with y^* the physical distance measured normal to the surface. ϕ is the amplitude of the disturbance

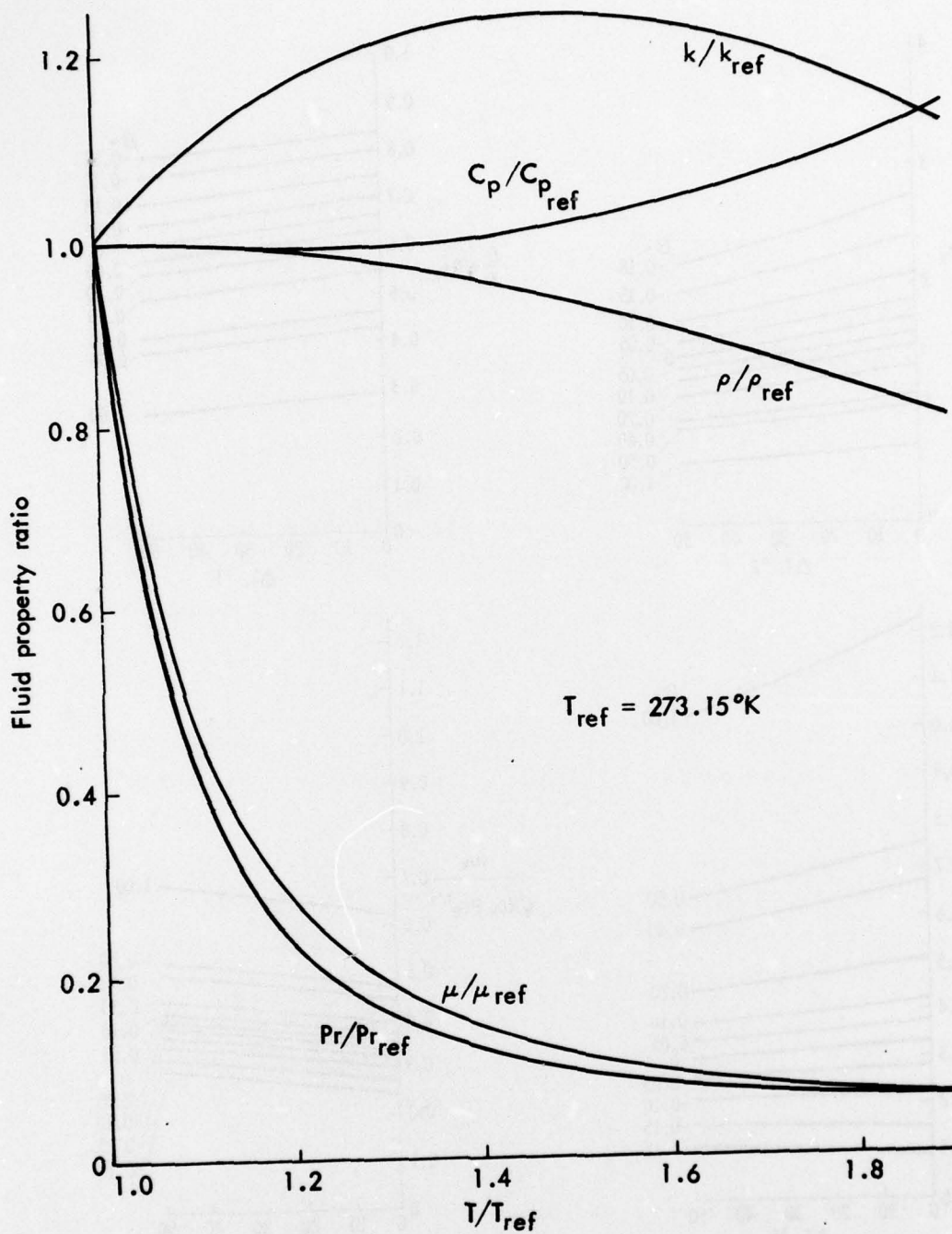


Fig. 1 — Dimensionless properties for water (2)

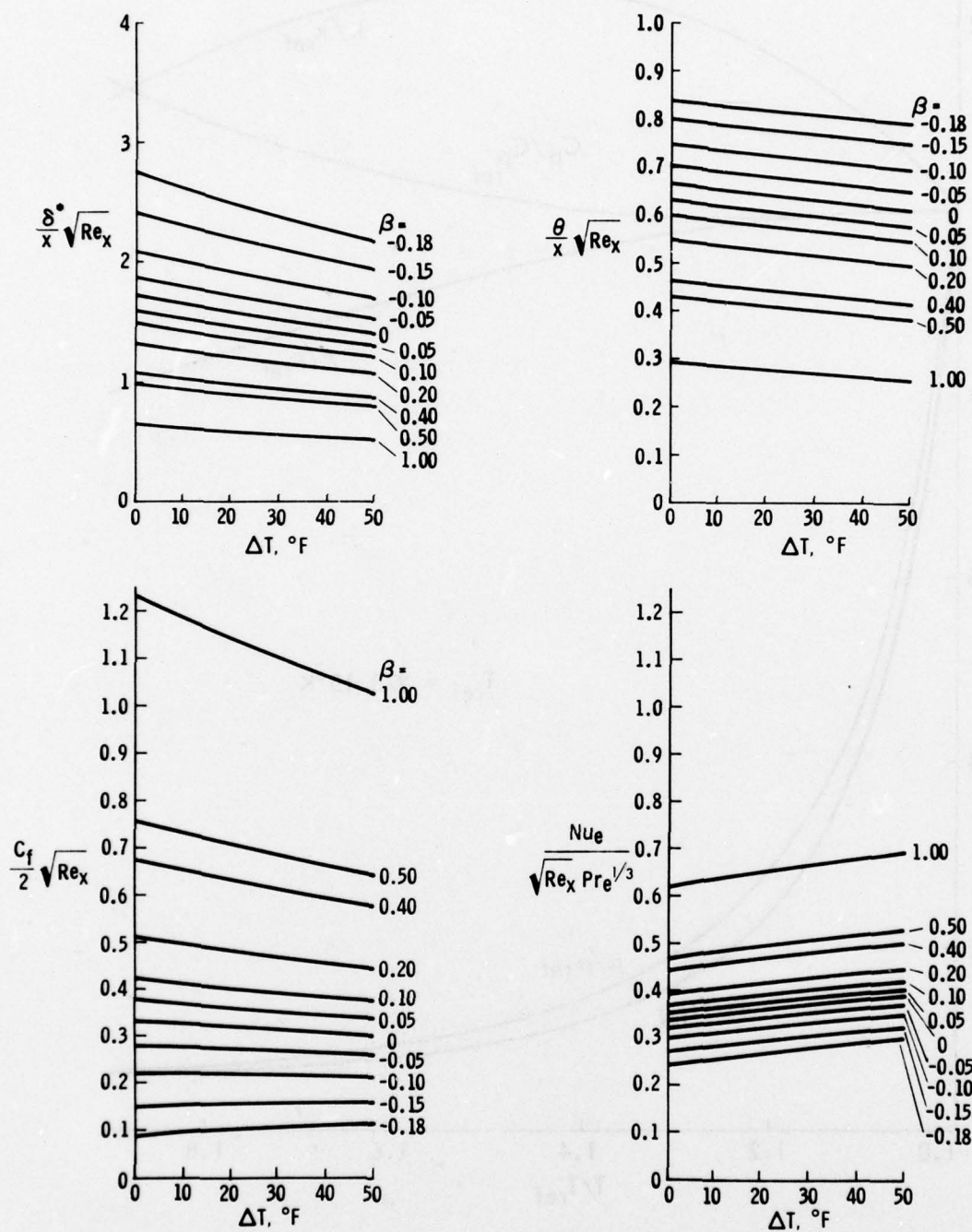


Fig. 2—Effect of heat transfer on the characteristics of laminar boundary layers in water ($T_e = 19.4^{\circ}\text{C}$, 67°F)

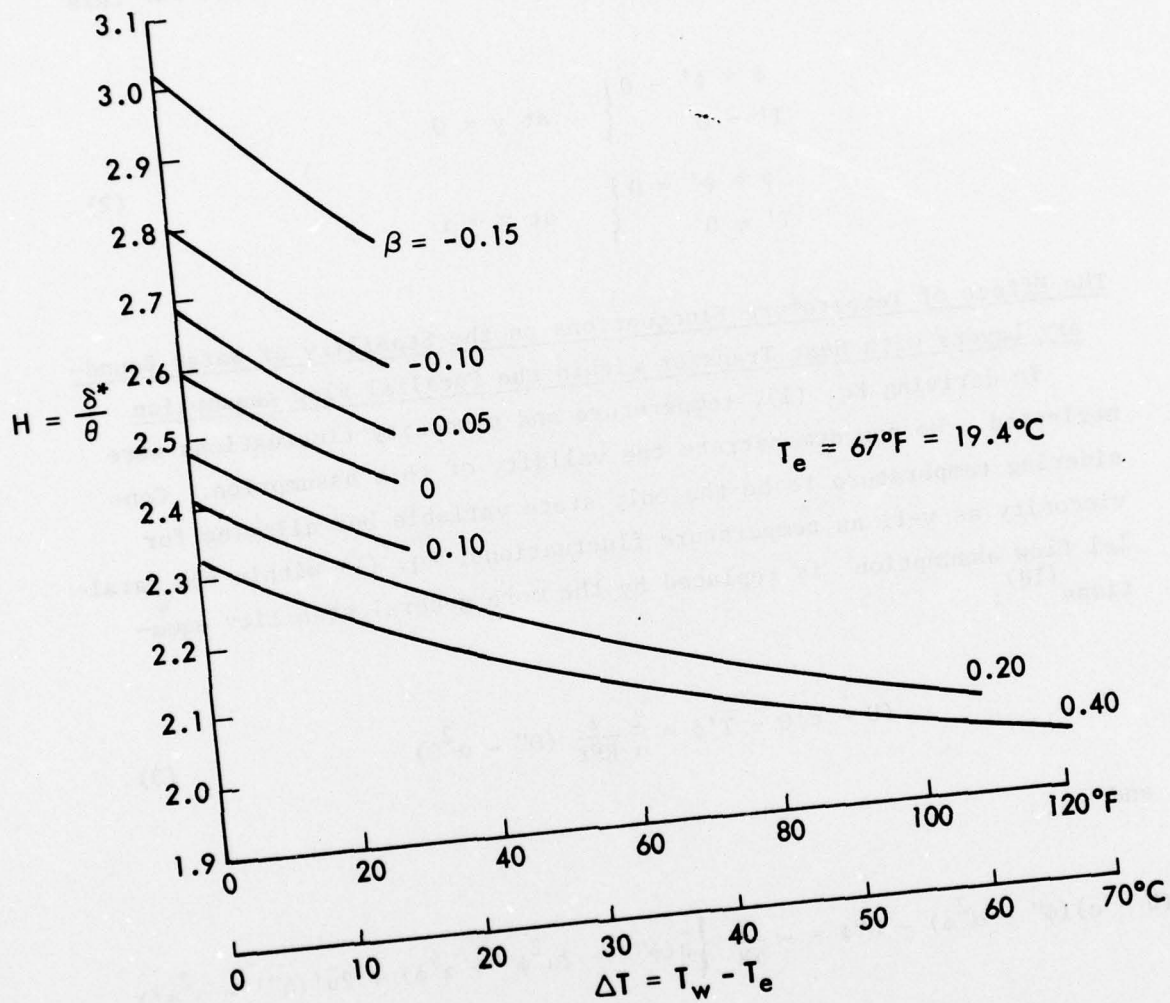


Fig. 3—Variation of the boundary-layer shape parameter for heated wedge flows in water

which is described by the stream function $\psi = \phi(y) e^{i\alpha(x-ct)}$ where α and c are complex quantities with αc , the frequency, taken real. In this case, the amplification is purely spatial; the amplification factor is of the form $e^{-\alpha i x}$. The proper boundary conditions for this problem are

$$\left. \begin{aligned} \phi &= \phi' = 0 \\ T' &= 0 \end{aligned} \right\} \quad \text{at } y = 0$$

$$\left. \begin{aligned} \phi &= \phi' = 0 \\ T' &= 0 \end{aligned} \right\} \quad \text{at } y \rightarrow 1$$
(2)

The Effect of Temperature Fluctuations on the Stability of Water Boundary Layers with Heat Transfer within the Parallel Flow Assumption

In deriving Eq. (1), temperature and viscosity fluctuations were neglected. We now demonstrate the validity of this assumption. Considering temperature to be the only state variable but allowing for viscosity as well as temperature fluctuations, Eq. (1) within the parallel flow assumption is replaced by the more general stability equations⁽¹⁸⁾:

$$(U - c)\theta - \bar{T}'\phi = \frac{i}{\alpha} \frac{1}{RPr} (\theta'' - \alpha^2 \theta) \quad (3)$$

and

$$\begin{aligned} (U - c)(\phi'' - \alpha^2 \phi) - U''\phi = & -\frac{i}{\alpha R} \left\{ \bar{\mu}(\phi'''' - 2\alpha^2 \phi'' + \alpha^4 \phi) + 2\bar{\mu}'(\phi''' - \alpha^2 \phi') \right. \\ & + \bar{\mu}''(\phi'' + \alpha^2 \phi) + \left[U'\bar{T}'' \frac{d^2 \bar{\mu}}{d\bar{T}^2} + U'(\bar{T}')^2 \frac{d^2 \bar{\mu}}{d\bar{T}^2} \right. \\ & + 2U''\bar{T} \frac{d^2 \bar{\mu}}{d\bar{T}^2} + U'''\frac{d\bar{\mu}}{d\bar{T}} + U' \frac{d\bar{\mu}}{d\bar{T}} \alpha^2 \left. \right] \theta \\ & \left. + 2U'' \frac{d\bar{\mu}}{d\bar{T}} \theta' + U' \frac{d\bar{\mu}}{d\bar{T}} \theta'' \right\} \end{aligned} \quad (4)$$

where Eq. (3) is the energy equation, Eq. (4) is the momentum equation, Pr is the mean Prandtl number, \bar{T} is the mean temperature, $T' = \Theta(y)e^{ia(x-ct)}$ is the temperature fluctuation, $\bar{\mu}$ is the mean viscosity, μ' is the viscosity fluctuation with $\mu \equiv \bar{\mu}(T) + (d\bar{\mu}/dT)T' + O(T'^2)$. A crude order of magnitude analysis of the new terms appearing in the right-hand side of the momentum equation [Eq. (4)] vis-à-vis the right-hand-side terms of Eq. (1) is made as follows. At a point in the boundary layer we assume $\phi \sim \Theta$, $\phi' \sim \Theta'$, $\phi'' \sim \Theta''$, $\alpha \sim 1/\delta$, $y \sim \delta$, $U \sim \bar{T}$, $d/dy \sim 1/\delta$. Now setting for example $a \equiv \bar{\mu}''\phi$, and $b \equiv U'\bar{T}''(d^2\bar{\mu}/dT^2)\Theta$, we find

$$(a/b) \approx \frac{\frac{\bar{\mu}}{\delta^2} \frac{\phi}{\delta^2}}{\frac{U}{\delta} \frac{\bar{T}}{\delta^2} \frac{\bar{\mu}''}{\bar{T}^2}} = \frac{1}{\delta} \gg 1 \quad (5)$$

Repeating the above analysis for other terms of Eq. (4), we find that the ratio of new terms in Eq. (4) arising from allowing for the viscosity and temperature fluctuations, to terms in Eq. (1) is always of order δ and hence can be neglected. These conclusions were recently confirmed by Lowell and Reshotko.⁽¹⁹⁾ These authors computed the stability of the flat-plate boundary layer in water with heat transfer. All fluid properties were allowed to vary with temperature in computing the mean flow as well as in computing its stability characteristics. These new results differed only slightly from the earlier results of Wazzan et al.,^(6,9) primarily due to the difference in ambient temperature (see below). Therefore, it appears that for water boundary layers, with moderate rates of heat transfer, it is sufficiently accurate to compute the effect of heat transfer on stability by using the single stability equation, namely, Eq. (1).

III. RESULTS

Equation (1) was solved for several wedge* flows, $\beta = -0.15, -0.10, -0.05, 0, 0.10, 0.20, 0.30, \text{ and } 0.40$, at $T_w - T_e = 0, 2.8, 11.1, 16.7, 27.8, 44.4, \text{ and } 66.7^\circ\text{C}$ ($0, 5, 20, 30, 50, 80, \text{ and } 120^\circ\text{F}$) with $T_e = 19.4^\circ\text{C}$ (67°F). The critical Reynolds number $Re_{x(\text{crit})}$ and the spatial amplification rates α_i as a function of Re_x were computed. The variation of $Re_{\delta^*(\text{crit})}$ with $(T_w - T_e)$ and with H are shown in Figs. 4 and 5. Re_{crit} initially increases with increasing ΔT ($\equiv T_w - T_e$) or with decreasing H . However, as ΔT is continuously increased, Re_{crit} attains a maximum value and decreases slightly (see also Fig. 7) as ΔT is further increased or H is further decreased. Since the effect of surface heating on boundary-layer stability is due primarily to the variation of viscosity with temperature, the results are thus dependent not only on the temperature difference $\Delta T = (T_w - T_e)$ but also on the temperature level, T_e . This is evident in Fig. 4, where previous results for $T_e = 15.6^\circ\text{C}$ (60°F) are compared with the present results for $T_e = 19.4^\circ\text{C}$ (67°F). Even this relatively slight change in ambient temperature results in an appreciable difference in the rate of change of viscosity with temperature, and consequently in the predicted stability.

DISCUSSION OF STABILITY RESULTS

Rayleigh theorems for inviscid instability state that for a boundary-layer flow, the necessary and sufficient condition for amplified and neutral inviscid instability is that $U''(y)$ must vanish somewhere in the

* Incompressible flow over wedges yields boundary layers which are "similar"--i.e., the boundary-layer velocity profile for a given wedge flow retains the same shape as the boundary layer develops (see, e.g., Ref. 4). Flow over a wedge having an included angle of $\pi\beta$ is characterized by an external velocity variation

$$U_e \sim x^{\frac{\beta}{2-\beta}}$$

where β is the Hartree parameter. Some common flows correspond as follows:

- $\beta = 1.0$ stagnation point
- $\beta = 0$ flat plate parallel to the flow
- $\beta = -0.1988$ boundary-layer separation

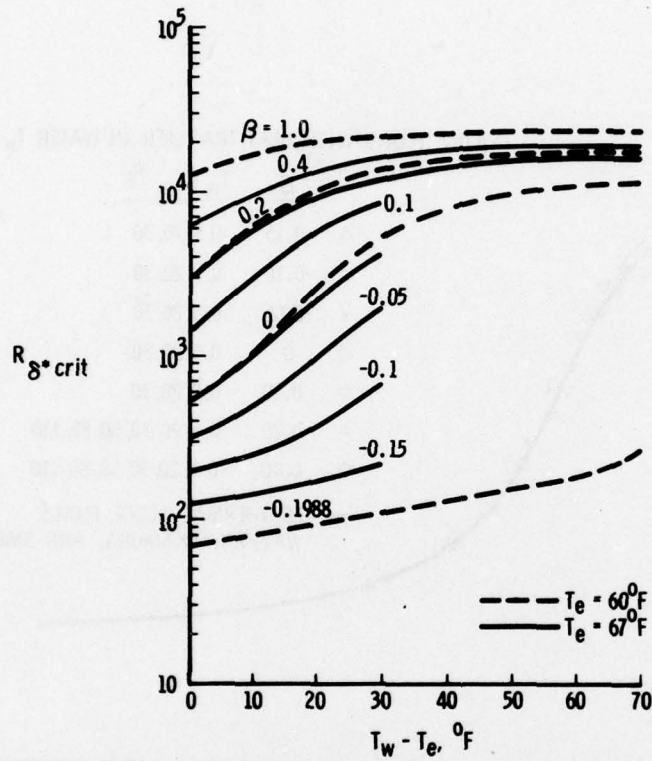


Fig. 4—Stability characteristics of laminar boundary layers in water

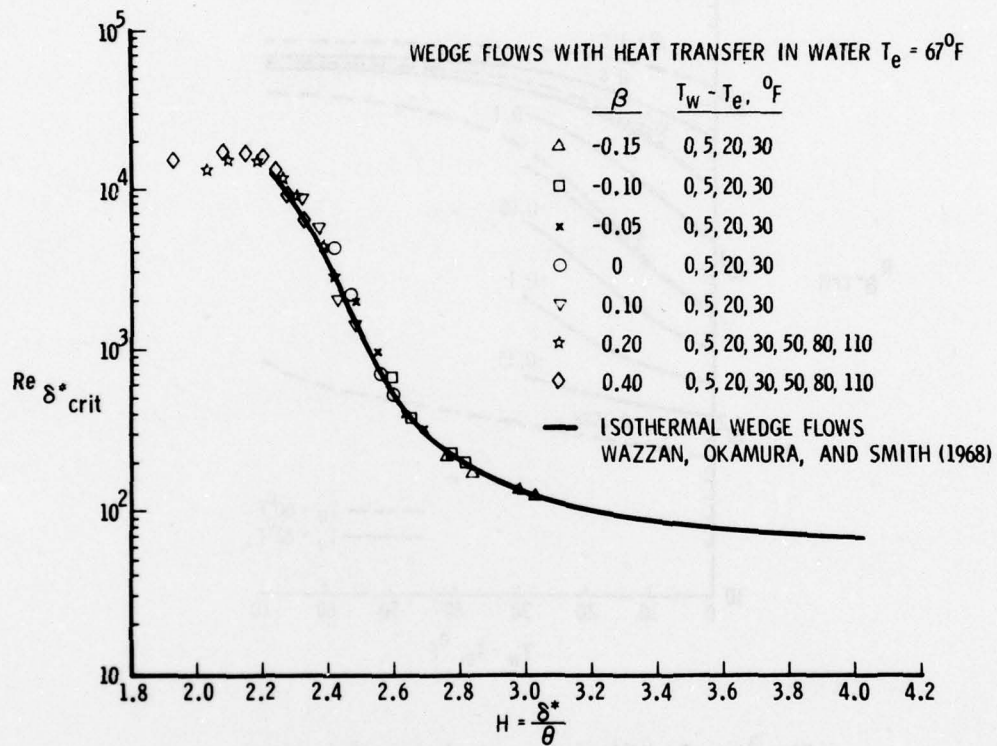


Fig. 5—Critical Reynolds number for heated wedge flows in water

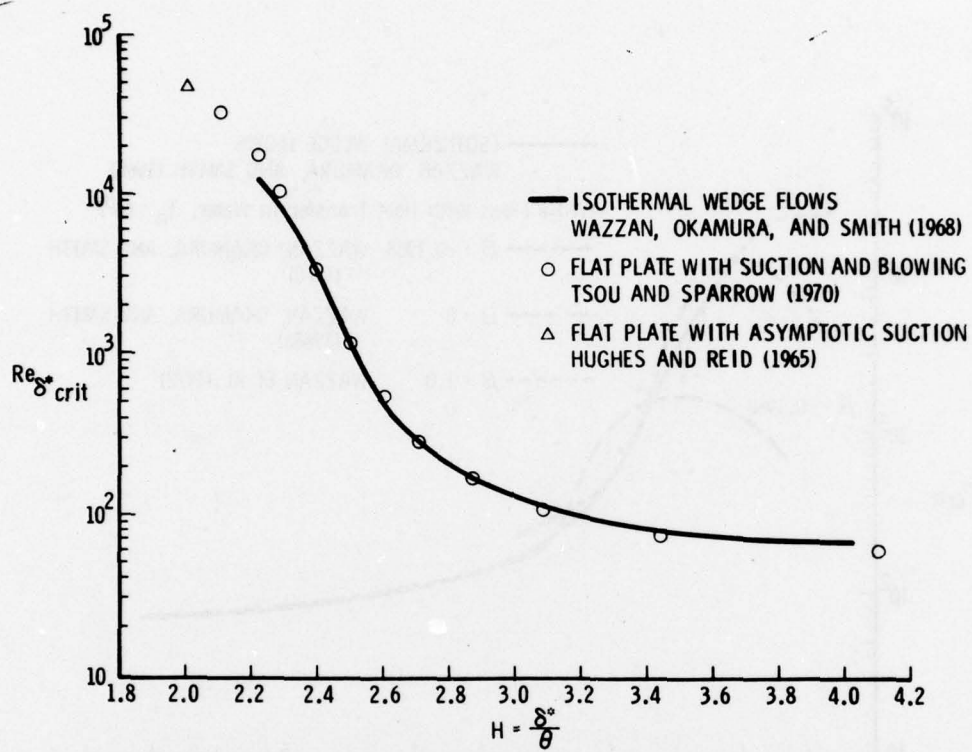


Fig. 6—Critical Reynolds number as a function of the boundary-layer shape parameter, isothermal wedge flows and flat plate with suction and blowing

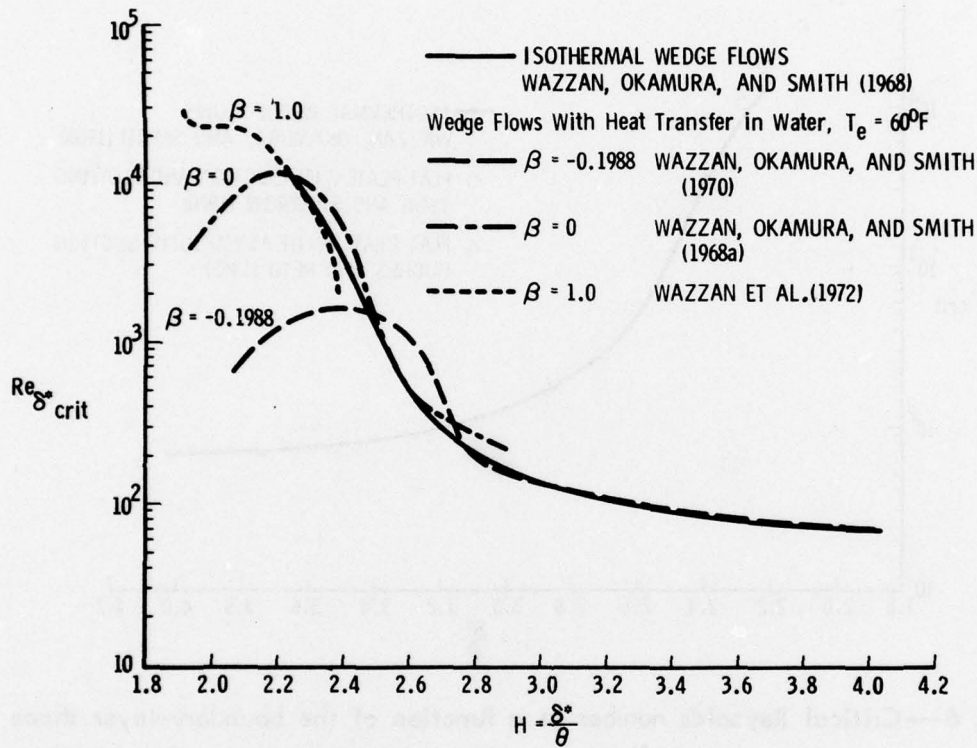


Fig. 7—Previous results for heated wedge flows in water

boundary layer, i.e., the mean velocity profile must be inflected. In addition, some correlation appears to exist between Re_{crit} and the location of the inflection points.^(9,20) In fact, in adiabatic liquid flows, $T_w = T_e$, Re_{crit} decreases as the location of the inflection point moves away from the wall. That is, in adiabatic flows, Re_{crit} decreases as β becomes more negative. In flows with zero or favorable pressure gradient ($\beta \geq 0$) where the profiles are not inflected, the boundary-layer characteristics of greatest importance to its stability characteristics are $U''(y)$ and to a lesser extent $U''(0)$.^(7, 16-18) In adiabatic flows it is known that for $\beta < 0$, where $U''(0) > 0$, Re_{crit} for $\beta < 0$ is smaller than Re_{crit} for $\beta > 0$. In fact, when a $U''(y)$ distribution is the result of pressure gradient effects only (adiabatic flows), a strong correlation exists between Re_{crit} and β [or $U''(y)$ or simply $U''(0)$] or the shape parameter H .^(7,16,17) The shape parameter provides a simple and convenient means of generalizing stability computations; Fig. 6 shows results for isothermal wedge flows⁽²¹⁾ and for suction^(22,23) and it is apparent that the effects of suction are essentially the same as favorable pressure gradient. Previous results for heated wedge flows^(6,7-9) are shown in Fig. 7; here also the effects of a boundary-layer modification by heating are qualitatively similar to the effects of pressure gradient except for relatively large temperature differences (e.g., over about 45°C or 80°F).

The critical Reynolds number thus exhibits a simple variation with $U''(0)$ or with H ; Re_{crit} increases as H decreases (Figs. 4, 5, and 6) or as $U''(0)$ decreases and/or becomes increasingly more negative. This dependence on U'' or H , in fact, is to be expected. An inspection of the Orr-Sommerfeld equation shows the boundary-layer characteristics that directly influence the eigen values, and hence Re_{crit} , are $U''(y)$ and $U(y)$, with $U''(y)$ being the dominant term. Therefore, it may be assumed that the heating of water boundary layers, which produces variations in H similar to those produced through the effect of pressure gradient alone, leads to increased stability and particularly to increasing critical Reynolds number. This assumption, however, is found to be only partially true (Figs. 5 and 7). These figures show that with initial heating H decreases and Re_{crit}

increases in agreement with the trend observed in Fig. 5. However, Figs. 5 and 7 show that although H decreases monotonically with increasing surface temperature, $Re_{\delta^*_{(crit)}}$ exhibits a maximum (at least for flows with $\beta \leq 1.0$). The difference in the variation of $Re_{\delta^*_{(crit)}}$ with H , when produced through the effect of pressure gradient alone (adiabatic flows) or through the combined effects of pressure gradient and surface heating, can be qualitatively understood through an examination of the Orr-Sommerfeld equation.

In the adiabatic case, only U'' and U appear in the Orr-Sommerfeld equation and hence the monotonic variation of $Re_{\delta^*_{(crit)}}$ with U'' or H . In the nonadiabatic case, the small disturbance equation is a modified Orr-Sommerfeld equation that contains not only U'' and U but also μ , μ' , and μ'' . This alters the nature of the problem in *two* ways. In the adiabatic case the pressure gradient affects $Re_{\delta^*_{(crit)}}$ mainly through the mean velocity term U'' (which can be represented by some function of H), whereas in the nonadiabatic case heating affects $Re_{\delta^*_{(crit)}}$ not only through the mean velocity U'' term (which can still be represented by some function of H) but also through the terms μ , μ' , and μ'' that appear in the modified Orr-Sommerfeld equation. Second, the nature of the eigen function ϕ (and consequently all eigen values and properties depending on the eigen values such as Re_{crit}) is different in the two cases. In the adiabatic case, the differential equation for ϕ (the Orr-Sommerfeld equation) includes only the function ϕ and the even derivatives ϕ'' and ϕ'''' , whereas in the nonadiabatic case, the differential equation for ϕ (the modified Orr-Sommerfeld equation) includes not only ϕ and the even derivatives ϕ'' and ϕ'''' but also the odd derivatives ϕ' and ϕ''' . Therefore, in the case of heating, although U'' or H still characterizes the boundary layer, neither present a complete relationship between the mean flow and the eigen function ϕ , and hence $Re_{\delta^*_{(crit)}}$. Physically, this may be interpreted as follows: with heating, initially H decreases rapidly indicating a decrease in momentum loss (Fig. 3) and the stability, e.g., $Re_{\delta^*_{(crit)}}$, increases. However, with still increased heating rates,

H continues to decrease but at a much slower rate (Fig. 3). In the meantime, μ , μ' , and μ'' continue to vary appreciably with increasing T_w . In fact μ , which has a destabilizing effect,^(6,8) continues to decrease monotonically with heating, whereas μ' and μ'' , and in particular $\mu'(0)$ and $\mu''(0)$, which have a stabilizing effect,⁽⁸⁾ reverse their trend in variation with temperature (change from increasing with T_w to decreasing with T_w) near the temperature where Re_{crit} exhibits a maximum for wedge flows with $\beta < 1.0$. Therefore, for high heating rates the variation of H with T_w becomes negligible and the variation of μ , μ' , μ'' with T_w dominates the effect of heating on Re_{crit} . Therefore, at high heating rates it is expected that $Re_{\delta^*_{(crit)}}$ will exhibit a maximum with T_w (Fig. 4 of this study and Fig. 1 of Ref. 8).

In the case of $\beta = 1.0$, the maximum in $Re_{\delta^*_{(crit)}}$ with T_w is not observed (Fig. 7) because the unstable zone (region contained within the neutral curve) is rather limited, and in the initial stages of heating when H is fast decreasing with T_w the unstable zone is fast approaching a point. In fact, at just about the temperature when the variation of H with T_w begins to slow down considerably and μ , μ' , and μ'' begin to dominate, the unstable region shrinks to zero and the flow becomes totally stable.

In spite of this discussion on the relative importance of H and/or the μ , μ' , and μ'' to the stability characteristics of a given boundary layer, Figs. 5, 6, and 7 show that over a large range of H values, a decrease in H results in increased stability, e.g., increasing $Re_{\delta^*_{(crit)}}$.

LINEAR STABILITY THEORY OF TWO-DIMENSIONAL DISTURBANCES AND TRANSITION

Of more practical importance than the variation of Re_{crit} with surface temperature and pressure gradient is the effect of these two parameters on transition. Although the transition process is complex and involves nonlinear processes in the final stages of breakdown to turbulence, much insight into the process of transition and its dependence on, e.g., heat transfer and pressure gradient can be gained from a study of the effect of these parameters on linear instability characteristics, such as Re_{crit} and the spatial amplification rates, local

and integrated values, particularly in the case of slowly amplifying boundary layers.

Because of the three-dimensionality of turbulence, early workers tended to ignore the role of two-dimensional linear amplification mechanisms, better known as Tollmien-Schlichting mechanisms or TS mechanisms. Schubauer and Skramstad⁽¹⁰⁾ and Liepmann⁽¹²⁾ verified, however, the features of the TS mechanism in flat-plate flow. On the other hand, Emmons⁽²⁴⁾ verified the existence of three-dimensional turbulent spots prior to transition. Criminale and Kovasznay⁽²⁵⁾ and Brooke⁽²⁶⁾ demonstrated, for various oblique TS waves, that localized areas of initially intensified disturbances should develop with strong two-dimensional features in the linear regime.

Reconciliation between early TS amplification and the final three-dimensionality of turbulence was achieved when transition was recognized to begin with linear TS amplification and to terminate with turbulent spots and wedges overcoming the mean laminar flow.⁽²⁷⁾ This model was reinforced when the qualitative effects of cooling, heating, suction, pressure gradient, Mach number, etc., on the stability of TS waves (theoretical studies and experimental observations) and on transition (experimental observations) were often found to be parallel.

Freestream Turbulence--Boundary-Layer Interaction

Disturbances that may excite or feed TS waves include:^(16,28,29)

- I. Temperature--density--entropy mode
- II. Vorticity--turbulence mode
- III. Sound mode

Mode I interacts with the boundary layer because of the growth of the boundary layer in the freestream direction. This mode is therefore not important in instability studies within the framework of the *parallel* flow assumptions. Mode II can disturb the boundary layer across stream lines because as it enters the layer it becomes distorted and stretched.⁽³⁰⁾ Some measurements by Hall⁽³¹⁾ and by Klebanoff⁽³²⁾ suggest that a boundary layer exhibits a variable receptivity towards freestream vorticity fluctuations. Aside from these observations, the effects of Mode II on TS waves are virtually unknown. According to Obremski et al.,⁽¹⁶⁾ sound of frequency ω_s , Mode III, excites

regular, coherent TS waves of the same frequency but different wave length, and the boundary-layer has a non-zero receptivity to acoustic disturbances with frequency in the modified TS range. Furthermore, when the primary acoustic frequency falls in the TS susceptibility region, ω_{TS} , the onset of transition can be dramatically changed.⁽³³⁾ Since the susceptible dimensional frequency band ω_{TS}^* scales primarily with U_∞^2/ν , i.e.,

$$\omega_{TS} = \frac{\omega^* \nu}{U_\infty^2} \quad (\text{dimensionless}),$$

a change in freestream velocity U_∞ (or a change in unit Reynolds number U_∞/ν) would change the onset of transition. Schubauer and Skramstad⁽¹⁰⁾ and Spangler and Wells⁽³⁴⁾ in subsonic flow, and Kendall⁽³⁵⁾ in supersonic flow, verified the strong influence sound has on transition; the elimination or reduction of sound sources was found to greatly increase the transition Reynolds number. [The importance of sound on transition in water boundary layers is yet to be demonstrated.] Aside from these observations on the effect of sound on transition, the process through which parts of the freestream sound energies become internalized as growing TS waves is not well understood.

Assessment of Transition

A disturbance growing according to linear instability theory sooner or later reaches a state where (1) the linear theory ceases to be valid, and nonlinear processes commence; (2) the boundary layer becomes locally turbulent--turbulent spots are formed and grow and increase in number; and (3) these spots spread into the neighboring laminar flow until the mean flow becomes fully turbulent. Therefore, satisfactory assessment of the beginning of transition for approximately two-dimensional boundary layers requires at least three elements:⁽¹⁶⁾ (a) adequate knowledge of the input disturbance and the corresponding boundary-layer receptivity; (b) knowledge of the development of the mean profiles and access to their stability characteristics; (c) information on the length of the nonlinear processes and secondary instability as dependent on pressure gradient,

heat transfer, etc. Since the information required in element (a) often is not available in the literature, one usually characterizes a disturbance in terms of the ratio $a[\omega, x(1), x(2)]$ of its amplitude A , $a \equiv A_{x(2)}/A_{x(1)}$, at two locations, $x(1)$ and $x(2)$.

According to Klebanoff et al.⁽³²⁾ stage (1) is reached, for a flat plate, when the rms velocity fluctuation u' in the boundary layer reaches $(u'/U)_{\max} = .015$, but the first appearance of turbulence spots is expected at $(u'/U)_{\max} = 0.2$. That is, beyond the onset of nonlinearity an amplification factor of 10 to 15 times ($\approx e^{2.5}$) is required.⁽¹⁶⁾ Liepmann⁽¹²⁾ hypothesized that at the breakdown to turbulence the Reynolds stress $\tau = -\rho \overline{u'v'}$, due to the amplified fluctuations u' , becomes comparable in magnitude to the maximum mean laminar shear stress, $\tau_L = \mu \partial u / \partial y$ in the boundary layer. The ratio τ / τ_L is given by

$$\tau / \tau_L = \frac{2}{c_{fL}} \left\{ kb(u'_n / U_e)^2 [a(x)]^2 \right\}_{\max}$$

where u'_n = disturbance amplitude at neutral point

$$b = v' / u'$$

$$\text{and } k = \overline{u'v'} / uv$$

where u and v are the velocities in the x and y direction, respectively. For a given frequency ω^* the amplification a is given by

$$a(x, \omega^*) = \exp \left[-R_L \int_{x_n}^x (\alpha_1 / R) (U_e / U_\infty) dx \right]$$

where here x is made dimensionless by division by the characteristic length L .

Smith⁽¹³⁾ reduced Liepmann's criterion for transition to an explicit dependence on the laminar skin friction coefficient, the disturbance input at the neutral point x_n , and the total amplification ratio $(A_{x(t)} / A_{x(n)})$,

where n refers to the neutral point and t to the transition point. Smith studied available transition data for attached boundary layers where the freestream turbulence level was low. Assuming linear theory valid up to the transition point, Smith showed that the ratio of the disturbance amplitude at transition $A_{x(t)}$ to that at the neutral point $A_{x(n)}$ is given by $(A_{x(t)}/A_{x(n)}) \equiv a(x_t, \omega_t^*) = e^9$. Later, more accurate calculations⁽¹⁴⁾ showed $(A_{x(t)}/A_{x(n)}) \equiv e^{10}$. In any event, since in the nonlinear zone the amplification to transition is $\approx e^{2.5}$, we find that for boundary layers with low freestream disturbance levels the linear TS amplification of about $e^{7.5}$ *does control* to a large extent the major part of the development of the disturbance to the beginning of transition and that *element (b)* of the transition process (paragraph 1 of section on "Assessment of Transition") appears, at least in this case, to *dominate* elements (a) and (c).

In spite of the dominant role of element (b), however, the role of element (a) remains extremely important. For example, when Spangler and Wells⁽³⁴⁾ minimized sound disturbances in their measurements of transition in a low-speed boundary-layer channel, their $R_{x(t)}$ exceeded five millions! These results, where a mixture of vorticity and sound disturbances is present, cannot be predicted using any of the presently available forecasting techniques.⁽³⁶⁾ Hence a knowledge of the receptivity of the boundary layer to vorticity and sound disturbances is needed for further progress, and much attention needs to be given element (a). In the absence of such information and in view of the fact that the TS mechanism may describe, in the absence of the effects of surface roughness, vibration, and sound, the substantial growth of disturbances up to the emergence of the final three-dimensional turbulent spots and wedges and the beginning of transition, it is not unreasonable to employ, for the present, linear theory in bracketing the Reynolds number at the beginning of transition for two-dimensional and axisymmetric boundary-layer flows (in axisymmetric flows x is replaced by s , the distance measured along the body surface).

Therefore, in certain boundary-layer flows where the linear mechanism dominates the growth of disturbances to transition, the transition Reynolds number can be bounded by $Re_{x(crit)}$ on the lower side and by $Re_{x(e^9)}$ on the upper side. Computations of the values of $Re_{x(e^9)}$ for heated wedge flows

are shown in Fig. 8 as a function of temperature difference and the trends are seen to be similar to those for the critical Reynolds number. Computations for both the critical and predicted transition Reynolds numbers are shown as a function of the shape parameter in Fig. 9.

A plot of $Re_{x(crit)}$ and $Re_{x(e^9)}$ vs H for two-dimensional wedge flows with and/or without heating (as shown in Fig. 9) can perhaps be used as a guideline in bracketing $Re_{x(trans)}$ on a body of revolution, for example. This can be accomplished by computing for the body of revolution H vs Re_x . If this locus of Re_x vs H falls between the two curves labeled $Re_{x(crit)}$ and $Re_{x(e^9)}$ in Fig. 9, the flow over the body may be considered to be completely laminar. If the locus of Re_x vs H crosses the $Re_{x(e^9)}$ curve, the boundary layer may be assumed to undergo transition at the Re_x of the intersection point. If the locus lies very close to the $Re_{x(crit)}$ curve, then the body is conservatively designed, whereas if the locus lies very close to the $Re_{x(e^9)}$ curve, then more heating and/or a more favorable pressure gradient would be desirable (to maintain laminar flow). These remarks, of course, may not hold completely since, as stated earlier, in heated flows H alone does not totally determine the stability, and hence the transition behavior of the body. Further confidence in this suggested analysis can be gained as measurements of $Re_{x(trans)}$ vs H become available and are used to check the validity of the trends indicated in Fig. 9.

The format of Fig. 9 has been chosen so as to allow easy application to specific problems. The path of boundary-layer development over a given shape on this diagram remains the same, since the shape parameter is only a function of the relative position on a (unheated) body. The path of boundary-layer development then simply moves up or down as the size and velocity of the body are changed.

As a test of these rather speculative comments, the wedge-flow computations have been compared with three cases of the development of non-similar boundary layers. The first of these is the development of a boundary layer in a heated tube; for this case, experimental data exist. (37) The boundary-layer development on the tube wall does not correspond to the similar type of boundary layer computed for the wedge flows; the local value of β increases parabolically from zero near the tube entrance to a

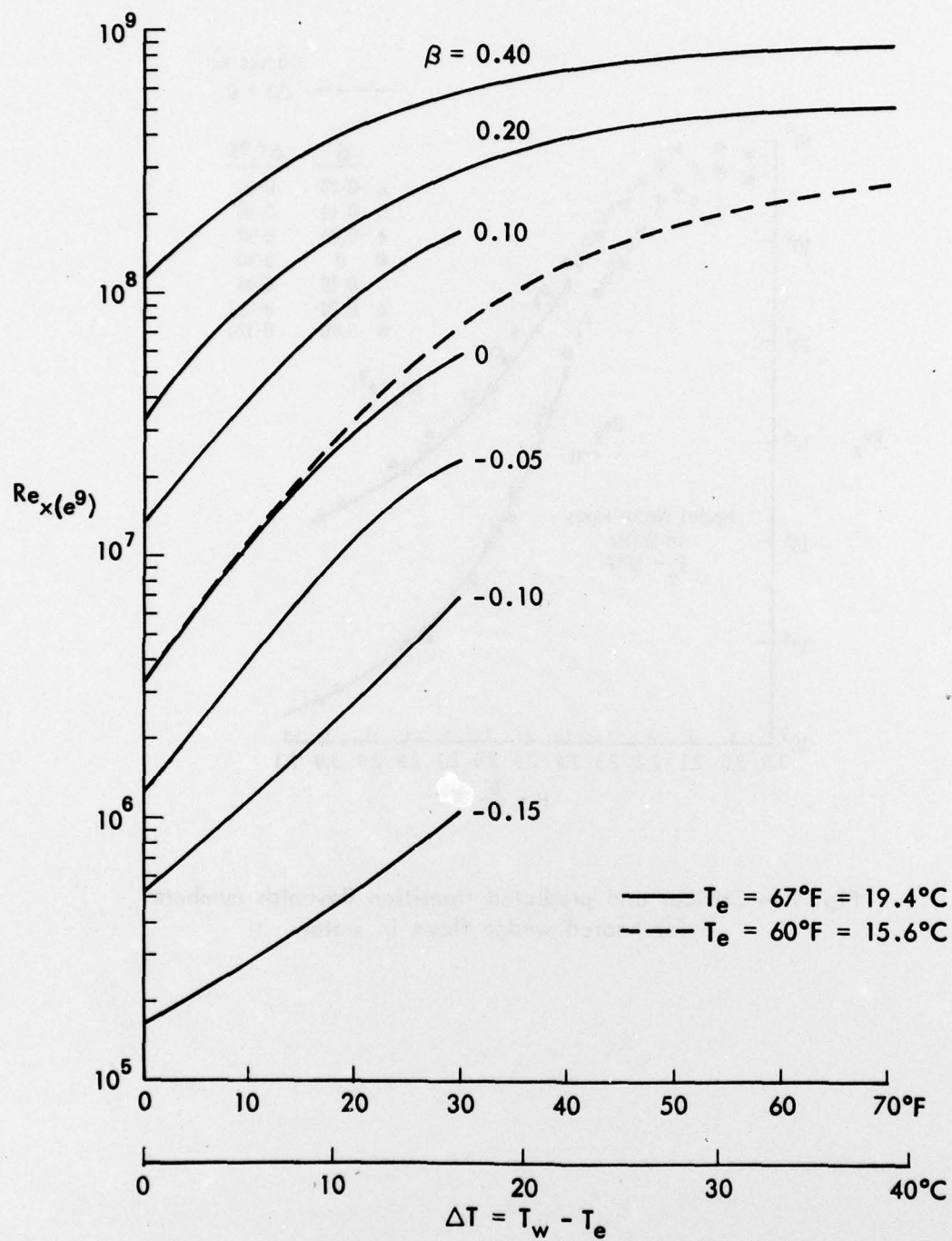


Fig. 8—Variation of $Re_x(e^9)$ for heated wedge flows in water

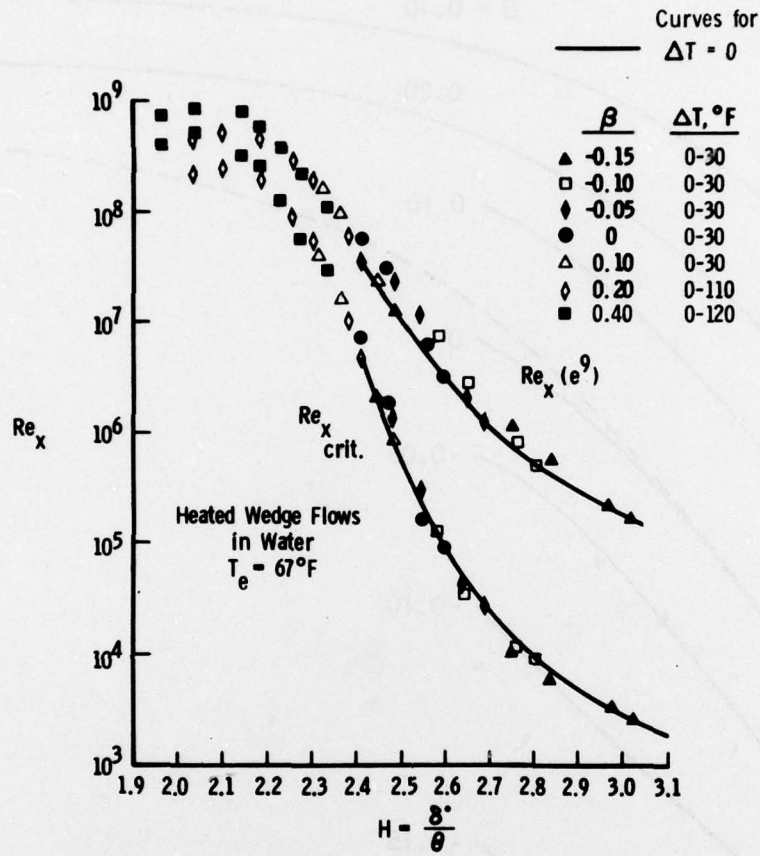


Fig. 9—Critical and predicted transition Reynolds numbers for heated wedge flows in water

positive value at the end of the tube. The path of the boundary-layer development on the Re_x -H plot of Fig. 9 would thus follow, not a vertical line as would be followed by the similar boundary layer on a wedge, but rather a line which is initially near vertical and curves slightly to the left corresponding to the increase in the local value of β (and decrease in the shape parameter H) as the flow proceeds along the pipe. The experimental measurements of Barker and Jennings⁽³⁷⁾ were made in a 6.1 m (20 ft) long, 0.1 m (4 in.) diameter tube using water at about 11°C (52°F). Their measurements correspond to conditions of velocity and wall temperature necessary to maintain a laminar boundary layer along the length of the tube. To attain these conditions, they found that it was necessary to follow carefully a "laminar path" in which velocity and wall temperature were simultaneously increased. Their data describing this "laminar path" are shown in Fig. 10 in comparison with wedge-flow computations for $Re_{x(e)}^9$ for the mean value of β in the tube (equal to two-thirds the value at the downstream end of the tube) and for an ambient temperature of 11°C (52°F). Agreement is remarkable up to a transition Reynolds number of $Re_x = 31 \times 10^6$ corresponding to a wall temperature about 5.5°C (10°F) above the ambient temperature. As the wall temperature is increased further, however, the measured values become increasingly less than the predicted ones. Whether this is due to buoyancy-induced secondary flows, to dirt deposits on the tube wall, or to downstream effects has yet to be determined.

The second case is a much more severe test of the wedge-flow computations. Computations of the boundary-layer development on a very blunt body of revolution^{(38)*} are shown in Fig. 11 against a background of the wedge-flow computation shown in Fig. 9.** The boundary-layer development for four unit Reynolds numbers over the unheated body are shown; also presented is the case of the body heated 5.5°C (10°F) above the ambient temperature. For this body, the departure from similarity is seen to be

*This shape was suggested to us by Professor A. J. Acosta of the California Institute of Technology.

**The isothermal wedge-flow lines of Fig. 9 are reproduced in Fig. 10 as dashed lines.

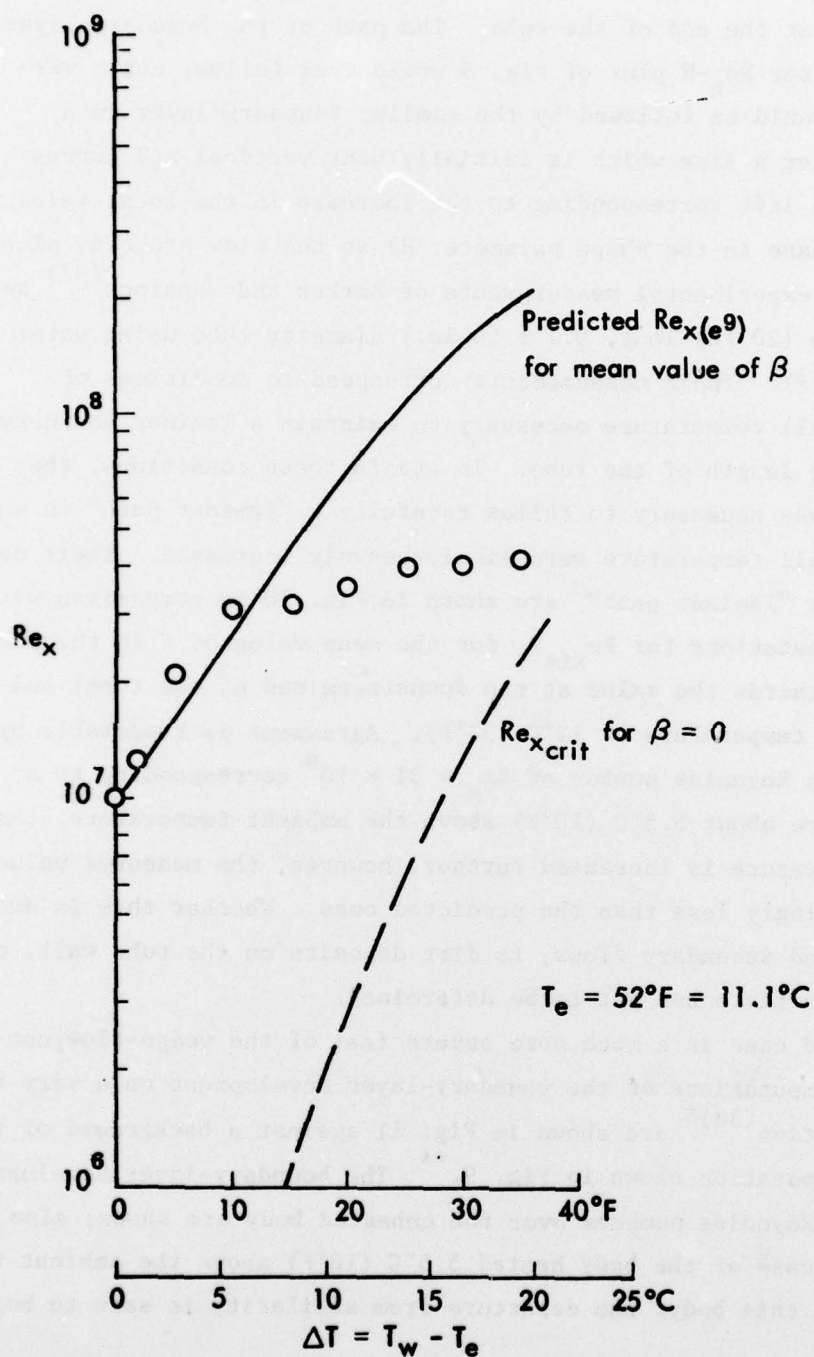


Fig. 10—Comparison of tube experiments⁽³⁵⁾ with prediction from wedge-flow computations

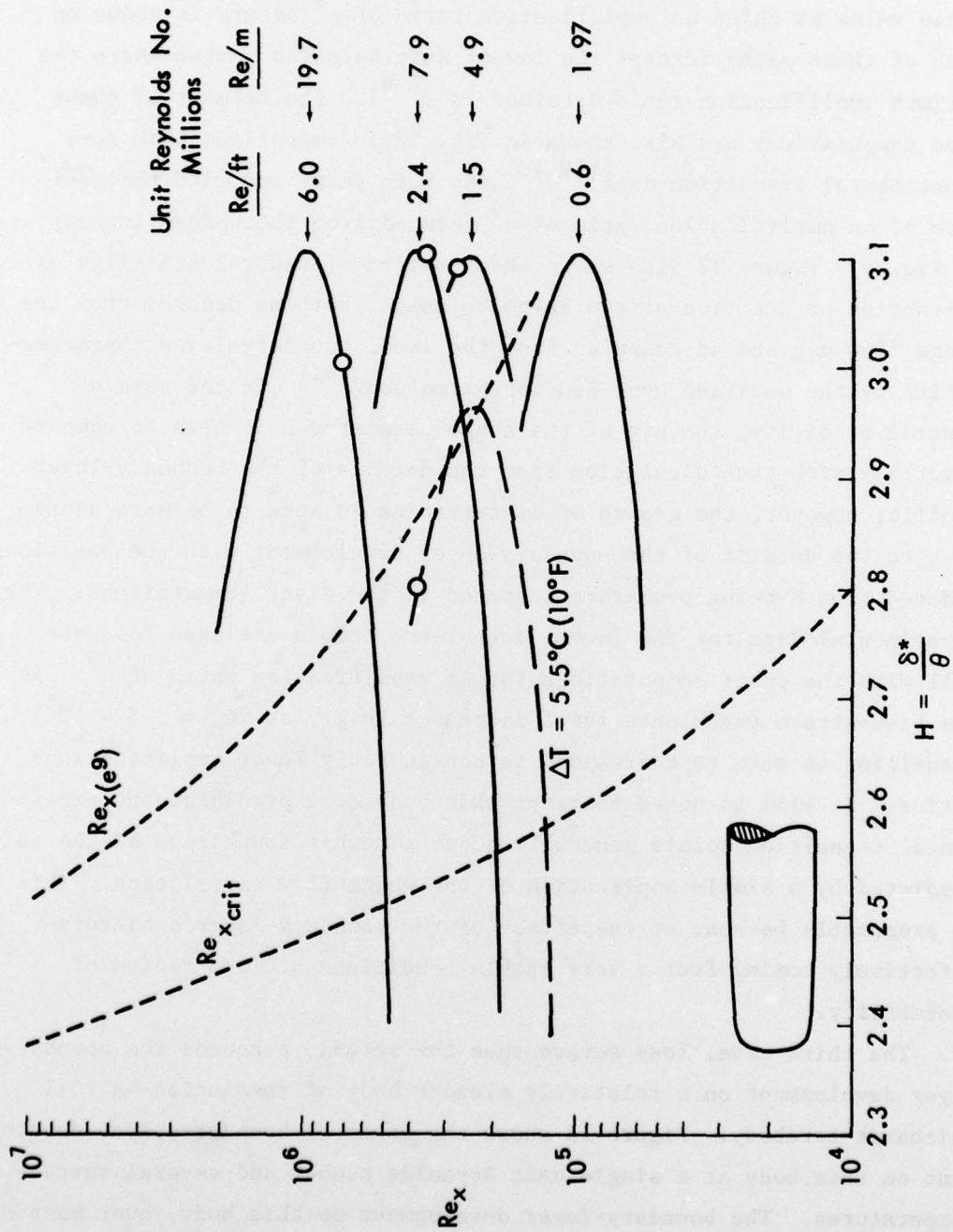


Fig. 11—Paths of boundary-layer development and predicted transition for a very blunt body. (36) Comparison with heated wedge-flow computations. Circles denote points where $Re_{x(e9)}$ is reached.

extreme. The initial path of boundary-layer development is initially almost horizontal on the Re_x -H plane with H increasing very rapidly as the boundary layer develops over the shoulder of the body; H then reaches a maximum and decreases rapidly on a higher horizontal path. The computed value at which an amplification ratio of e^9 occurs is shown on each of these paths (except the lowest unit Reynolds number where the maximum amplification ratio attained is $e^{7.8}$). The results of these same computations are also shown in Fig. 12 in comparison with some experimental transition data^(39,40) for this shape and with the position of an amplification ratio of e^9 deduced from the wedge-flow map of Fig. 9. Figure 12 also shows the position of neutral stability as a function of the free-stream Reynolds number both as deduced from the wedge flow map and as computed from the local boundary-layer characteristics by the modified Dunn-Lin approximation.⁽⁴¹⁾ In the case of neutral stability, the use of the shape parameter H is seen to compare favorably with the calculation from the details of the boundary-layer profile; however, the growth of disturbances is seen to be more sensitive to the details of the boundary-layer development with the positions deduced from H being premature compared to the exact computations. The experimental data for the lowest turbulence levels are seen to agree well with the exact computations for an amplification ratio of e^9 . As the free-stream turbulence level increases (e.g., at $Re_D = 2.5 \times 10^5$), transition is seen to correspond to considerably lower amplification ratios. It will be noted that for this body, the predicted and experimental transition points generally occur somewhat downstream of the point predicted by a simple application of the wedge-flow correlation. This is presumably because of the effect of the boundary-layer's history--effectively coming from a very stable condition into the region of instability.

The third case, less severe than the second, concerns the boundary-layer development on a relatively slender body of revolution--a 13:1 Reichardt forebody. Figure 13 shows the paths of boundary-layer development on this body at a single unit Reynolds number and several surface temperatures. The boundary-layer development on this body, over most of its length, more closely approximates the similar wedge flows. The

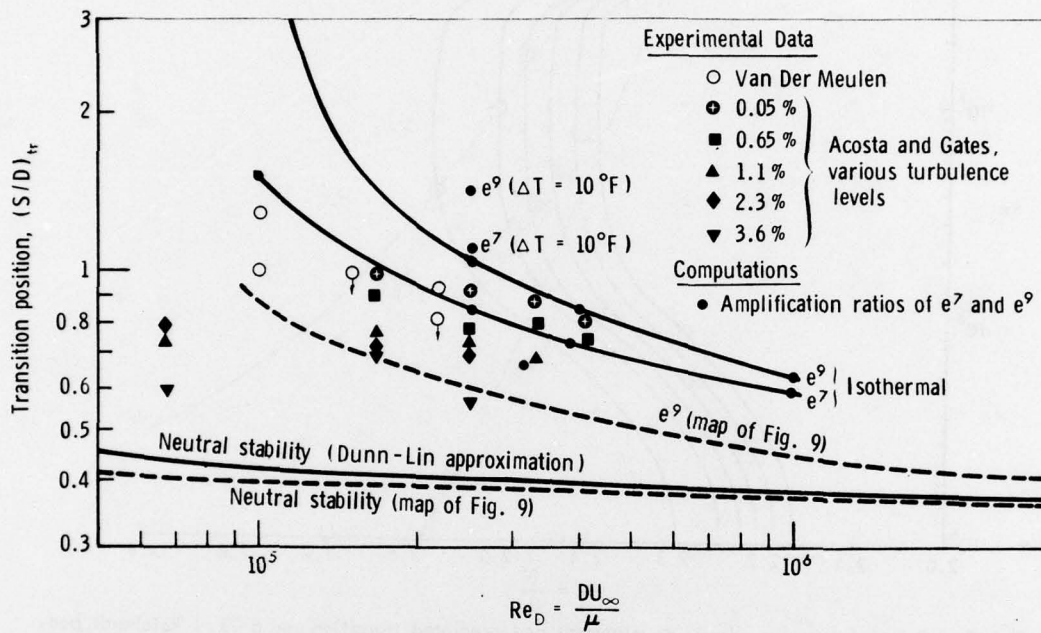


Fig. 12 — Comparison of computed neutral stability and predicted transition (e^9) with experimental transition data and with values deduced from the map of Fig. 9

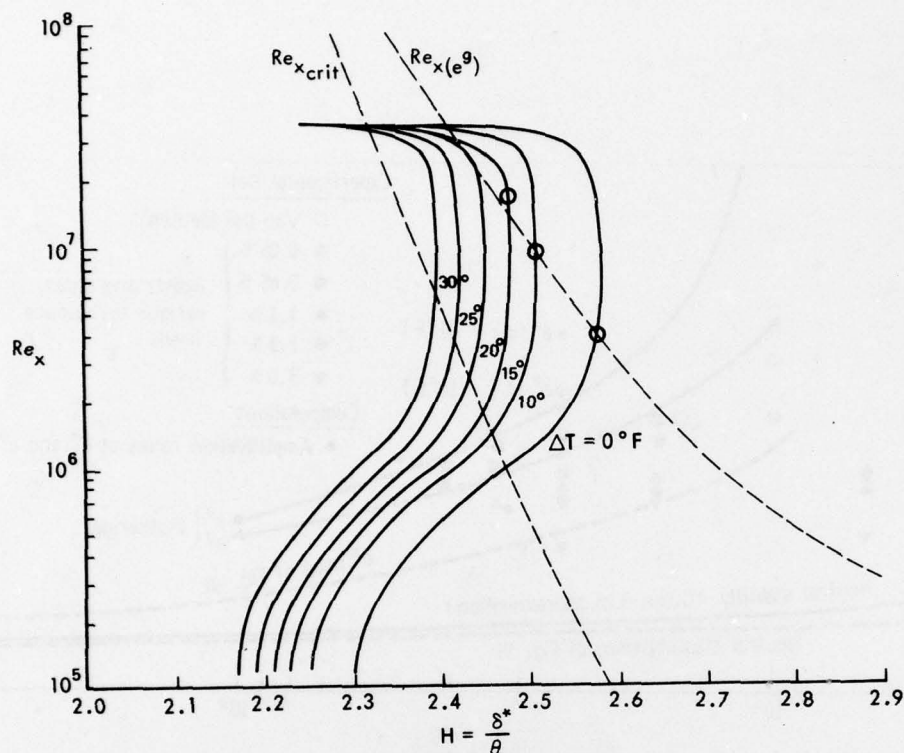


Fig.13 — Paths of boundary-layer development and predicted transition for a 13:1 Reichardt body. Comparison with heated wedge-flow predictions. Circles denote point where $Re_x(e^9)$ is reached.

computed positions of an amplification ratio of e^9 are shown on these curves and are seen to coincide closely with the wedge-flow computations for $Re_{x(e^9)}$.

These example comparisons appear to confirm the utility of the application of the wedge-flow results, and their "correlation" in terms of the shape parameter $H = \frac{\delta^*}{\theta}$, to estimation of boundary-layer stability and transition for non-similar flows. For near similar flows (Figs. 10 and 13) the wedge-flow results compare very well with experiment and exact computation. For an extreme departure from similarity (Figs. 11 and 12), the boundary-layer history must be taken into account. In general, keeping the shape parameter as low as possible over as great a range of Reynolds number as possible is desirable for maintaining an extended region of laminar flow.

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⇒ Appreciable drag reduction is possible if extended regions of laminar flow can be maintained. Although a variety of techniques for boundary-layer control have been explored, only recently has the powerful effect of heat transfer on the stability and transition of water boundary layers been realized. This report presents computational results for the stability and predicted transition characteristics of water boundary-layer "wedge" flows for various combinations of pressure gradient and heat transfer. Both the minimum critical Reynolds number and the predicted transition Reynolds number of these "similar" boundary layers increase as the surface temperature is increased above the ambient level. The interacting effects of pressure gradient and surface heating on stability and predicted transition may be approximately characterized by a boundary-layer shape parameter. ~~In order~~ to maintain an extended region of laminar flow, the boundary-layer development should follow a path in which the shape parameter is kept as low as possible over as great a range of Reynolds number as possible. (See also R-1752, R-1789, R-1863, R-1898, R-1966, R-2111, R-2164, R-2165, R-2209.) Refs. (Author)

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